Exam 1

CSc 75010: Theoretical Computer Science

Graduate Center of CUNY

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(Yellow version)

Do five of the following six problems. Write each answer on a separate piece of paper.

1. Define the following terms:

   (a) finite state automaton

   (b) regular language

   (c) Given a finite set $\Sigma$, define $\Sigma^*$

   (d) Given a string $s$, define $|s|$

   (e) Given string $s_1, s_2$, define $s_1 \circ s_2$

2. Find the error in the following proof that $2 = 1$.
   Consider the equation $a = b$. Multiply both sides by $a$ to obtain $a^2 = ab$. Subtract $b^2$ from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side, $(a+b)(a-b) = b(a-b)$, and divide each side by $(a - b)$, to get $a + b = b$. Finally, let $a$ and $b$ equal 1, which shows $2 = 1$.

3. Give the state diagrams of NFAs recognizing the following languages. In all cases the alphabet is $\Sigma = \{a, b, c, d, \ldots, x, y, z\}$, the 26 lowercase letters.

   (a) $\{w \mid w$ contains the substring $yellow\}$

   (b) $\{w \mid w$ is of even length or ends with the substring $bye\}$

4. Prove that the class of regular languages is closed under the star operator.

5. Let $\Sigma = \{0, 1, +, =\}$. Prove that the following language is not regular:

   $$ADD = \{x = y + z \mid x, y, z \text{ are binary integers, and x is the sum of y and z}\}$$

6. Give context-free grammars generating the following languages. The language is $\Sigma = \{a, b\}$.

   (a) $\{w \mid w$ starts and ends with the same symbol$\}$

   (b) the complement of the language $\{a^n b^n \mid n \geq 0\}$