

Exam 1
CSc 75010: Theoretical Computer Science
Graduate Center of CUNY
4 October 2002
(Sample Exam)

Do five of the following six problems. Write each answer on a separate piece of paper.

1. Define the following terms:

- (a) regular expression
- (b) Pigeonhole Principle
- (c) stack
- (d) Given a string s , define $|s|$
- (e) Given finite sets Σ_1, Σ_2 , define $\Sigma_1 \circ \Sigma_2$

2. Find the error in the following proof that all horses are the same color.

CLAIM: In any set of h horses, all horses are the same color.

PROOF: By induction on h .

Basis: For $h = 1$. IN any set containing just one horse, all horses clearly are the same color.

Induction step: For $k \geq 1$, assume that the claim is true for $h = k$ and prove that it is true for $h = k + 1$. Take any set H of $k + 1$ horses. We show that all the horses in this set are the same color. Remove one horse from this set to obtain the set H_1 with just k horses. By the induction hypothesis, all the horses in H_1 are the same color. Now replace the removed horse and remove a different one to obtain the set H_2 . By the same argument, all horses in H_2 are the same color. Therefore, all the horses in H nyst be the same color, and the proof is complete.

3. Give the state diagrams of NFAs recognizing the following languages. In all cases the alphabet is $\Sigma = \{a, b, c, d, \dots, x, y, z\}$, the 26 lowercase letters.

- (a) $\{w \mid w \text{ contains the substring } help\}$
- (b) $\{w \mid w \text{ is of length at least 2 and an even numbers of } z\text{'s}\}$

4. Prove that the class of regular languages is closed under the union operator.

5. Prove that the following language is not regular:

$$\{w \mid w \in \{0, 1\}^* \text{ is not a palindrome}\}$$

6. Give context-free grammars generating the following languages:

- (a) $\{w \mid \text{the length of } w \text{ is odd and its middle symbol is } 0\}$
- (b) The complement of the language $\{a^n b^n \mid n \geq 0\}$