Exam 1
CSc 75010: Theoretical Computer Science
Graduate Center of CUNY
4 October 2002
(Pink version)

Do five of the following six problems. Write each answer on a separate piece of paper.

1. Define the following terms:
   (a) regular expression
   (b) Pigeonhole principle
   (c) Given string s_1, s_2, define $s_1 \circ s_2$
   (d) Given finite sets Σ_1, Σ_2, define $\Sigma_1 \cup \Sigma_2$
   (e) Given finite sets Σ_1, Σ_2, define $\Sigma_1 \cap \Sigma_2$

2. Find the error in the following proof that $2 = 1$.
   Consider the equation $a = b$. Multiply both sides by a to obtain $a^2 = ab$. Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side, $(a+b)(a-b) = b(a-b)$, and divide each side by $(a-b)$, to get $a + b = b$. Finally, let a and b equal 1, which shows $2 = 1$.

3. Give the state diagrams of NFAs recognizing the following languages. In all cases the alphabet is $\Sigma = \{a, b, c, d, \ldots, x, y, z\}$, the 26 lowercase letters.
   (a) $\{w \mid w$ contains the substring $pink\}$
   (b) $\{w \mid w$ is of odd length and begins with the substring $hi\}$

4. Prove that the class of regular languages is closed under the concatenation operator.

5. Prove that the following language is not regular:
   $$\{0^m 1^n \mid m \neq n\}$$

6. Give context-free grammars generating the following languages for $\Sigma = \{0, 1\}$:
   (a) $\{w \mid$ the length of w is odd$\}$
   (b) $\{w \mid w$ contains twice as many 1s as 0s$\}$