

Exam 1
CSc 75010: Theoretical Computer Science
Graduate Center of CUNY
4 October 2002
(Green version)

Do five of the following six problems. Write each answer on a separate piece of paper.

1. Define the following terms:

- (a) finite state automaton
- (b) regular language
- (c) stack
- (d) Given finite sets Σ_1, Σ_2 , define $\Sigma_1 \cap \Sigma_2$
- (e) Given finite sets Σ_1, Σ_2 , define $\Sigma_1 \circ \Sigma_2$

2. Find the error in the following proof that $2 = 1$.

Consider the equation $a = b$. Multiply both sides by a to obtain $a^2 = ab$. Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side, $(a + b)(a - b) = b(a - b)$, and divide each side by $(a - b)$, to get $a + b = b$. Finally, let a and b equal 1, which shows $2 = 1$.

3. Give the state diagrams of NFAs recognizing the following languages. In all cases the alphabet is $\Sigma = \{a, b, c, d, \dots, x, y, z\}$, the 26 lowercase letters.

- (a) $\{w \mid w \text{ contains the substring } green\}$
- (b) $\{w \mid w \text{ is of even length or ends with the substring } bye\}$

4. Prove that the class of regular languages is closed under the star operator.

5. Prove that the following language is not regular:

$$\{0^n 1^m 0^n \mid m, n \geq 0\}$$

6. Give context-free grammars generating the following languages for $\Sigma = \{0, 1\}$:

- (a) $\{w \mid \text{the length of } w \text{ is odd}\}$
- (b) $\{w \mid w \text{ contains more 1s than 0s}\}$