

Exam 1
CSc 75010: Theoretical Computer Science
Graduate Center of CUNY
4 October 2002
(Blue version)

Do five of the following six problems. Write each answer on a separate piece of paper.

1. Define the following terms:

- (a) regular language
- (b) stack
- (c) Given a finite set Σ , define Σ^*
- (d) Given a string s , define $|s|$
- (e) Given finite sets Σ_1, Σ_2 , define $\Sigma_1 \circ \Sigma_2$

2. Find the error in the following proof that $2 = 1$.

Consider the equation $a = b$. Multiply both sides by a to obtain $a^2 = ab$. Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side, $(a + b)(a - b) = b(a - b)$, and divide each side by $(a - b)$, to get $a + b = b$. Finally, let a and b equal 1, which shows $2 = 1$.

3. Give the state diagrams of NFAs recognizing the following languages. In all cases the alphabet is $\Sigma = \{a, b, c, d, \dots, x, y, z\}$, the 26 lowercase letters.

- (a) $\{w \mid w \text{ contains the substring } blue\}$
- (b) $\{w \mid w \text{ is of even length and begins with the substring } hi\}$

4. Prove that the class of regular languages is closed under the union operator.

5. Prove that the following language is not regular:

The complement of $\{0^n 1^n \mid n \geq 0\}$

6. Give context-free grammars generating the following languages. Both are over the alphabet $\Sigma = \{a, b\}$.

- (a) $\{w \mid w = w^R, \text{ that is } w \text{ is a palindrome}\}$
- (b) The set of strings with twice as many a 's as b 's.