Name: ______________________________

Question I.1	
Question I.2	
Question I.3	
Question I.4	
Question I.5	
Question II.1	
Question II.2	
Question II.3	
Total	

Final Exam

CMP 416/685: Computability Theory
Lehman College– CUNY, 27 May 2004

Directions:
- **Write each answer on a separate piece of paper.**
- **Undergraduates: do any 5 of the problems.**
- **Graduates: Do 5 of the problems.**
At least 2 problems must be chosen from Part II.
- **If you complete more than 5 questions,**
the highest scores will be used to calculate your grade.

Part I: Undergraduate Questions

1. (a) Define the following terms:
   - Turing-recognizable
   - decidable
   - mapping reducible ($\leq_m$)

   (b) If a language A is decidable is its complement, $\bar{A}$, decidable? Why or why not?

   (c) If a language A is Turing-recognizable and $A \leq_m B$, is B Turing-recognizable? Why or why not?

   (d) If $A \leq_m B$ and $B \leq_m C$, then is $A \leq_m C$? Why or why not?

2. (a) State the pumping lemma for regular languages.

   (b) State the pumping lemma for context-free languages.

   (c) Prove $A = \{a^n b^n c^n \mid n \geq 0\}$ is not regular.

   (d) Is A context-free? If yes, give a context-free grammar for it. If no, use the pumping lemma to show that's it not.

3. Given the context-free grammar:

$$S \rightarrow aSb \mid SS \mid \epsilon$$

   (a) Convert this grammar to an equivalent pushdown automata (PDA).

   (b) Convert this grammar to one in Chomsky Normal Form.

4. Let $\Sigma = \{a, b\}$. For each machine below, give full implementation-level details (that is, write down every state of the machine and a state diagram).

   (a) Build a Turing Machine that halts if and only if the input string begins with 00.

   (b) Build a Turing machine doubles its input (that is, if the input number is x, the output would be $2x$.)

   (c) Build a Turing machine that given any input, it never halts.

5. (a) State the Halting Problem.

   (b) What is the Diagonalization method? Explain.

   (c) Use the Diagonalization method to show that the Halting problem is undecidable.

Part II: Graduate Questions

1. (a) Show that the class of regular languages is closed under intersection.
   (b) Show, by induction, that $(1+2+\cdots+n)^2 = 1^3 + 2^3 + \cdots n^3$.
2. Describe the **errors** in the following proofs:
   (a) Find the error in the following proof that $2 = 1$.
   Consider the equation $a = b$. Multiply both sides by a to obtain $a^2 = ab$. Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side, $(a+b)(a-b) = b(a-b)$, and divide each side by $(a-b)$, to get $a + b = b$. Finally, let a and b equal 1, which shows $2 = 1$.
   (b) Find the error in the follow proof that 0^*1^* is not regular.
   The proof is by contradiction. Assume that 0^*1^* is regular. Let p be the pumping length for 0^*1^* given by the pumping lemma. Chose s to be 0^p1^p. You know that s is a member of 0^*1^*, but for the proof that $\{0^n1^n \mid n \geq 0\}$ is not regular, s could not be pumped. Thus, you have a contradiction. So, 0^*1^* is regular.
3. Show that if A is Turing-recognizable and $A \leq_m \bar{A}$, then A is decidable.