

Name: _____

Exam 2

CMP 416/685: Computability Theory
Lehman College— CUNY, 29 April 2004

Directions:

- Write each answer on a separate piece of paper.
 - Undergraduates: do any 5 of the problems.
 - Graduates: Do 5 of the problems.
- At least 2 problems must be chosen from Part II.
- If you complete more than 5 questions, the highest scores will be used to calculate your grade.

Question I.1	
Question I.2	
Question I.3	
Question I.4	
Question I.5	
Question II.1	
Question II.2	
Question II.3	
Total	

Part I: Undergraduate Questions

- (a) Define the following terms:
 - regular language
 - context free language
 - decidable language(b) Is every regular language context-free? Why or why not?
(c) Is every decidable language context-free? Why or why not?
- Show, by induction, that $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$.
- (a) Write the context free grammar (CFG) for the following:
$$\{w \mid w \text{ starts and ends with the same symbol}\}$$
(b) Is your grammar in Chomsky Normal Form? If so, why? If not, write an equivalent grammar that is.
- Let $\Sigma = \{a, b\}$. For each machine below, give full implementation-level details.
 - Build a Turing Machine that halts if and only if the input string begins with 00. (That is, show that the language $\{w \mid w \text{ contains a } 0\}$ is Turing-recognizable.
 - Build a Turing machine adds one to its input (that is, if the input number is x , the output would be $x + 1$.)
- (a) State the pumping lemma for context-free languages.
(b) Use the pumping lemma to show that the language $B = \{a^n b^n c^n \mid n \geq 0\}$ is not context free.

Part II: Graduate Questions

- Show that the collection of decidable languages is closed under complementation.
- Let $B_n = \{a^k \mid \text{where } k \text{ is a multiple of } n\}$. Using induction on n , show that for $n \geq 1$, the language B_n is regular.
- (a) Give a context-free grammar that generates the language
$$A = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and either } i = j \text{ or } j = k\}$$
(b) Is your grammar ambiguous? Why or why not?