Name: _____

Exam 1
CMP 416/685: Computability Theory
Lehman College– CUNY, 28 February 2008

Directions:
• Write each answer on a separate piece of paper.
• Undergraduates: do any 5 of the problems.
• Graduates: Do 5 of the problems.
At least 2 problems must be chosen from Part II.
• If you complete more than 5 questions,
the highest scores will be used to calculate your grade.

Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Question 6	
Question 7	
Question 8	
Question 9	
Question 10	
Total	

Part I: Undergraduate Questions

1. Define the following terms:
   (a) finite state automaton
   (b) regular language
   (c) Given a finite set Σ, define Σ^*
   (d) Given a string s, define $|s|$
   (e) Given finite sets Σ_1, Σ_2, define $\Sigma_1 \circ \Sigma_2$

2. Give the state diagrams of deterministic finite state automata (**DFAs**) recognizing the following languages. In all parts, $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.
   (a) $\{w \mid w$ has at least three a's and has at least two b's$\}$
   (b) $\{w \mid w$ starts with an a and has at most one b$\}$
   (c) $\{w \mid w$ has even number of a's and one or two b's$\}$

3. Give the state diagrams for nondeterministic finite state automata (**NFAs**) with the specified number of states recognizing the following languages. In all parts, $\Sigma = \{0, 1\}$.
   (a) $\{w \mid w$ contains the substring 0101$\}$ with five states
   (b) The language 0^* with one state
   (c) The language $\{0\}$ with two states

4. Give the state diagrams of deterministic finite state automata (**DFAs**) recognizing the following languages. In all parts, $\Sigma = \{0, 1\}$.
   (a) $\{w \mid w$ begins with 1 and ends with a 0$\}$
   (b) $\{w \mid w$ contains exactly two 1's$\}$
   (c) the union of the two languages:
   $\{w \mid w$ begins with 1 and ends with a 0, or w contains exactly two 1's$\}$

5. Give the state diagrams for nondeterministic finite state automata (**NFAs**) recognizing the following languages. In all parts, $\Sigma = \{a, b\}$.

   (a) $\{w \mid w$ contains the substring `abab`$\}$

   (b) $\{w \mid w$ does not contains the substring `abab`$\}$

   (c) $\{w \mid w$ is a string in `(abab)`$^*\}$

6. Give the state diagrams for nondeterministic finite state automata (**NFAs**) recognizing the following languages. In all parts, $\Sigma = \{0, 1\}$.

   (a) `(00)`*

   (b) `(101)` $\cup$ `1`*

   (c) `(00)`*`((101)` $\cup$ `1`*`)`

Part II: Graduate Questions

7. Give the state diagrams for nondeterministic finite state automata (**NFAs**) recognizing the following languages. In all parts, the alphabet is $\Sigma = \{a, b, c, d, \ldots, x, y, z\}$, the 26 lowercase letters.

   (a) $\{w \mid w$ contains the substring *yellow*$\}$

   (b) $\{w \mid w$ is of even length or ends with the substring *bye*$\}$

8. In certain programming languages, comments appear between delimiters such as `/#` and `#/`. Let C be the language of all valid delimited comment strings. A member of C must begin with `/#` and end with `#/` but have no intervening `#/`. For simplicity, we'll say that the comments themselves are written with only the symbols `a` and `b`; hence the alphabet of C is $\Sigma = \{\texttt{a}, \texttt{b}, /, *\}$.

   (a) Give a DFA that recognizes C.

   (b) Give a regular expression that generates C

9. Prove that the class of regular languages is closed under the star operator.

10. Prove that every NFA can be converted to an equivalent one that has a single accept state.