Name: _____

Exam 1
CMP 416/685: Computability Theory
Lehman College– CUNY, 29 September 2016

Directions:
• Write each answer on a separate piece of paper.
• Undergraduates: do any 5 of the problems.
• Graduates: Do 5 of the problems.
At least 2 problems must be chosen from Part II.
• If you complete more than 5 questions,
the highest scores will be used to calculate your grade.

Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Question 6	
Question 7	
Question 8	
Question 9	
Question 10	
Total	

Part I: Undergraduate Questions

1. Define the following terms:
   (a) deterministic finite automata
   (b) nondeterministic finite automata
   (c) Given a finite set Σ, define Σ^*
   (d) Given finite sets Σ_1, Σ_2, define $\Sigma_1 \cup \Sigma_2$
   (e) regular language

2. Give the state diagrams of deterministic finite state automata (**DFAs**) recognizing the following languages. In all parts, $\Sigma = \{0, 1\}$.
   (a) $\{w \mid w$ starts with 00$\}$
   (b) $\{w \mid w$ contains exactly 3 1's$\}$
   (c) $\{w \mid w$ contains at most 3 1's$\}$

3. Give the state diagrams for nondeterministic finite state automata (**NFAs**) with the specified number of states recognizing the following languages. In all parts, $\Sigma = \{0, 1, 2, 3\}$.
   (a) $\{w \mid w$ contains the substring 3210$\}$ with five states
   (b) The language (02) with three states
   (c) The language $(02)^*$ with three states

4. Give the state diagrams of deterministic finite state automata (**DFAs**) recognizing the following languages. In all parts, $\Sigma = \{0, 1\}$.
   (a) $\{w \mid w$ begins with 0 and ends with 11$\}$
   (b) $\{w \mid w$ contains exactly one 1's$\}$
   (c) the union of the two languages:
   $\{w \mid w$ begins with 0 and ends with 11, or w contains exactly one 1's$\}$

5. Give the state diagrams for nondeterministic finite state automata (**NFAs**) recognizing the following languages. In all parts, $\Sigma = \{a, A\}$.

   (a) $\{w \mid w$ contains the substring `aAAa`$\}$

   (b) $\{w \mid w$ does not contains the substring `aAAa`$\}$

   (c) $\{w \mid w$ is a string in `(aAAa)`$^*\}$

6. Show that the following languages are not regular. In all parts, $\Sigma = \{0, 1\}$.

   (a) $\{w \mid w = 0^n 1^n, n \geq 0\}$

   (b) $\{w \mid w \neq 0^n 1^n, n \geq 0\}$ (i.e., the complement of the language in the first part).

Part II: Graduate Questions

7. Let $\Sigma = \{0, 1, +, =\}$ and

   $$ADD = \{x = y + z \mid x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}$$

   Show that ADD is not regular.

8. In certain programming languages, text appear between delimiters such as ‘ and ’. Let T be the language of all valid delimited text strings. A member of T must begin with ‘ and end with ’ but have no intervening ’. For simplicity, we'll say that the text themselves are written with only the symbols `a` and `b`; hence the alphabet of C is $\Sigma = \{\texttt{a}, \texttt{b}, \text{‘}, \text{’}\}$.

   (a) Give a DFA that recognizes T.

   (b) Give a regular expression that generates T

9. Prove that the class of regular languages is closed under concatenation.

10. Prove that every NFA can be converted to an equivalent one that has a single accept state.