Algorithmic Approaches for Biological Data, Lecture #16

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City University of New York
American Museum of Natural History

30 March 2016

Outline

- Networks & Graphs

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- Standard Representations: Adjacency Lists and Adjacency Matrices

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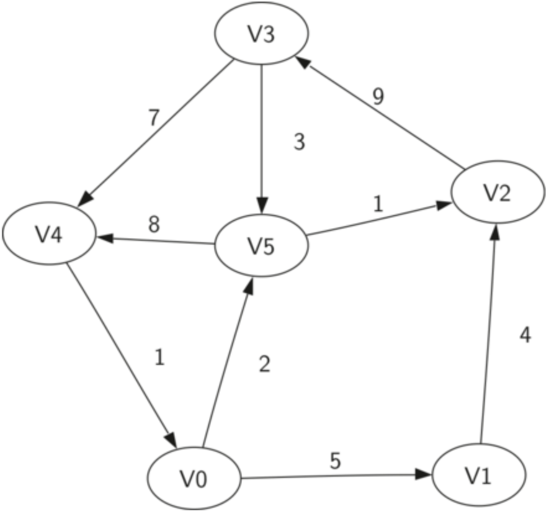
- Networks & Graphs
- Standard Representations: Adjacency Lists and Adjacency Matrices
- Reframing Biology Questions

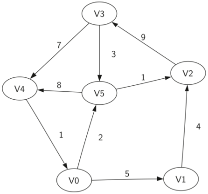
Networks & Graphs



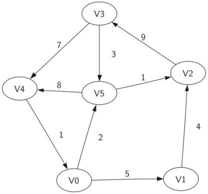
Problem Solving with Algorithms and Data Structures

Networks & Graphs

- Graphs (networks)

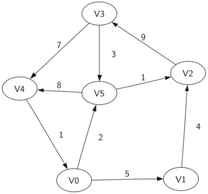


Networks & Graphs



- Graphs (networks) have vertices (nodes)

Networks & Graphs



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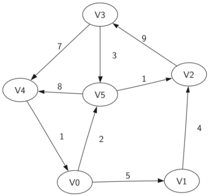

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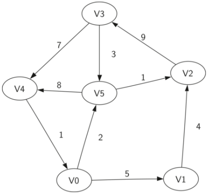
- Edges can have weights.

Networks & Graphs



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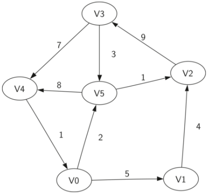

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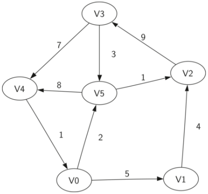

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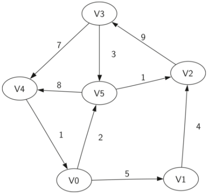

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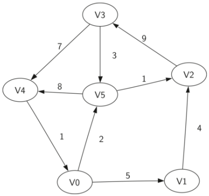
Networks & Graphs



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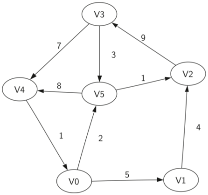
  - Since edges have a direction, called a directed graph.

Networks & Graphs



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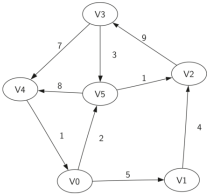
Networks & Graphs



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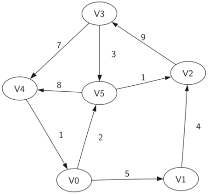

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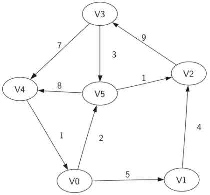

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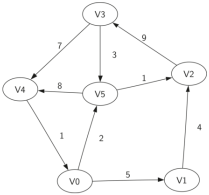

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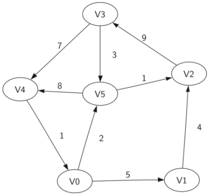

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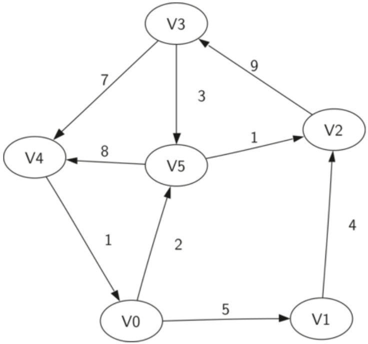
- A directed graph with no cycles is called a directed acyclic graph (DAG).

Representing Graphs in the Computer: Adjacency Matrix



	V0	V1	V2	V3	V4	V5
V0		5				2
V1			4			
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V3					7	3
V4	1					
V5			1		8	

Problem Solving with Algorithms and Data Structures

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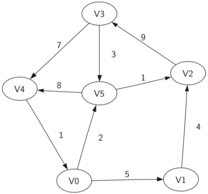

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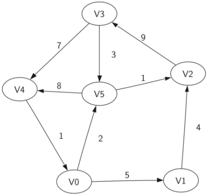

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- ```
  import numpy as np
  adjMatrix = np.zeros(6,6)
  adjMatix[0,1] = 5
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  ```
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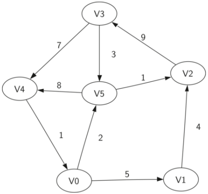

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  ```
- Need to keep track of the node names separately.
  ```

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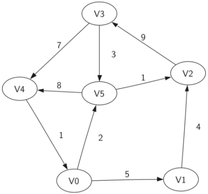

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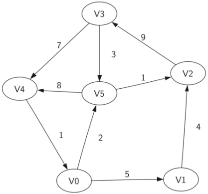

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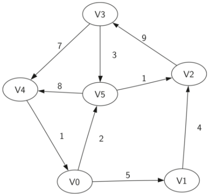

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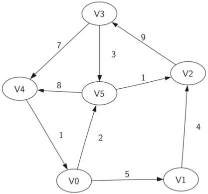

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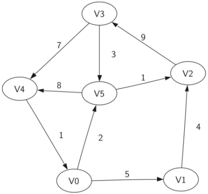

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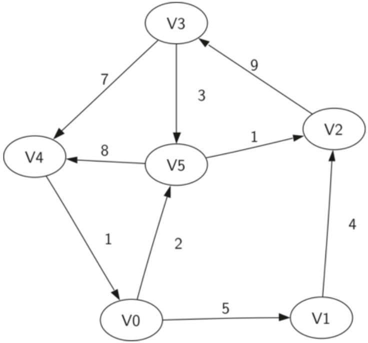
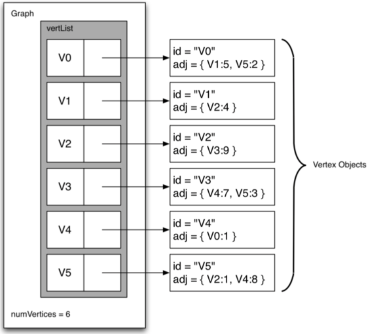
  - Checking if an edge occurs is quick.
  - Can check connectivity by matrix multiplication (explained in lab).
- Disadvantages:
  - Always the same size ($n \times n$) even if there are few edges.

Representing Graphs in the Computer: Adjacency List





Problem Solving with Algorithms and Data Structures

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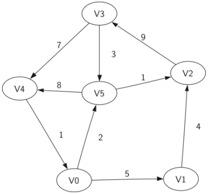

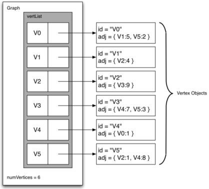

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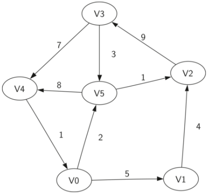

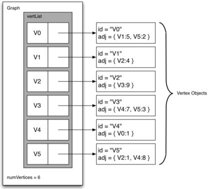

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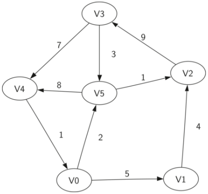

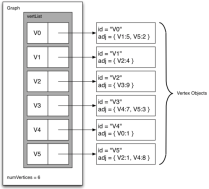

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```
- Can look up each list of adjacencies in the dictionary using the vertex label as the key.

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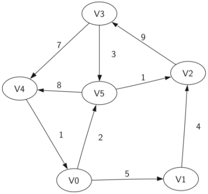
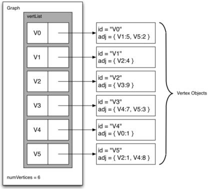
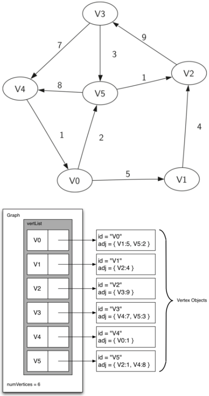




- Advantages:

Representing Graphs in the Computer: Adjacency List



- Advantages:
  - More space efficient for sparsely connected graphs

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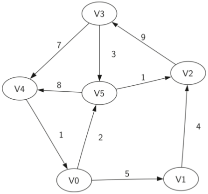

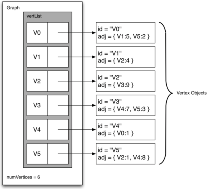

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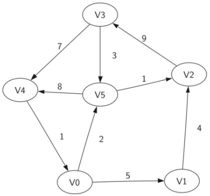
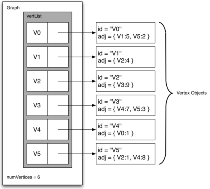
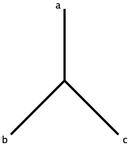




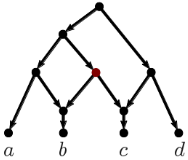
- Advantages:
  - More space efficient for sparsely connected graphs
- Disadvantages:
  - Could be costly to find adjacencies if a vertex has many neighbors.

In Pairs

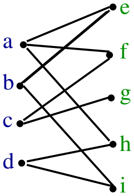
In pairs/triples, represent the following graphs in the computer:



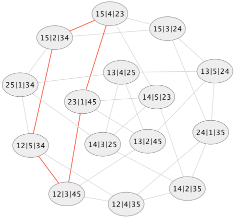
(1)



(2)

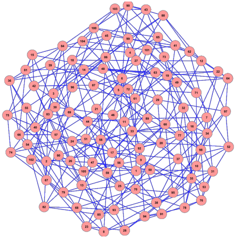


(3)



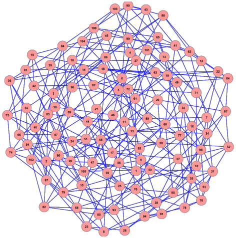
(4)

Recap



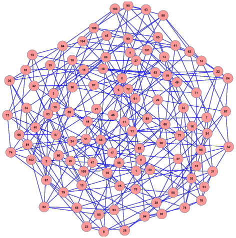
- Lab today: connectivity & storing trees

Recap



- Lab today: connectivity & storing trees
- Using `networkx` in lab today
  (for displaying graphs).

Recap



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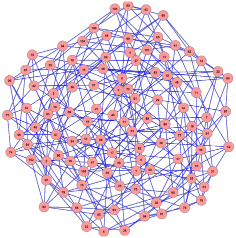
  (for displaying graphs).
- Email lab reports to `kstjohn@amnh.org`

Recap



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- Challenges available at `rosalind.info`