Algorithmic Approaches for Biological Data, Lecture #13

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Outline

- Recap: Lists and Arrays

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- Sorting:

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  - Insertion

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- Recursion: functions that call themselves

Recap: Lists

Creating lists: `a = []`

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del a[1]
del c[4:6]
```

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Deleting elements:

np arrays are immutable, but can do:

```
newA = np.delete(a, 4)
```

In Pairs

In pairs/triples, work out (and then try at the shell or `pythonTutor`):

1. What does the following code do?

```
a = [12,3,1,50,18,6,15,34]
print a, a[::-3]
print max(a), max(a[::-3])
```

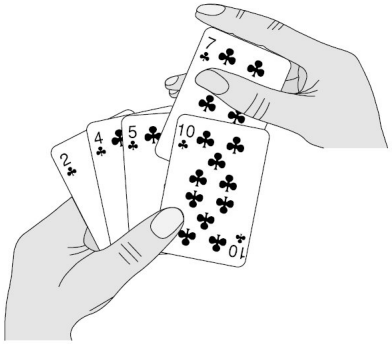
2. What does the following code do?

```
def mystery(a):
for i in range(1,len(a)):
    if a[i-1] > a[i]:
        a[i-1], a[i] = a[i], a[i-1]
a = [11,34,1,20,18,6,5,3]
print a
mystery(a)
print a
mystery(a)
print a
```

3. Describe how to find the largest card in a hand.
4. Describe how to sort a hand of cards.
5. What does the following code do?

```
def mystery2(a,b):
    c = []
    ai = 0
    bi = 0
    while ai < len(a) and bi < len(b):
        if a[ai] < b[bi]:
            c.append(a[ai])
            ai += 1
        else:
        c.append(b[bi])
        bi += 1
    if ai < len(a):
        c = c + a[ai:]
    if bi < len(b):
        c = c + b[bi:]
    return c
x = [1,3,5,6,7,9]
y = [2,4,8,10,12,15]
z = mystery2(x,y)
print z
```

Sorting



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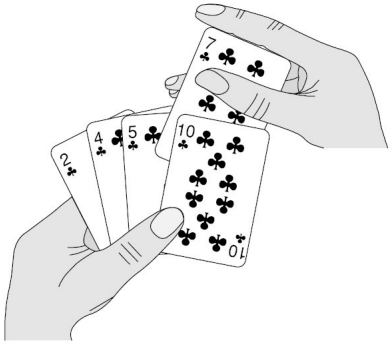

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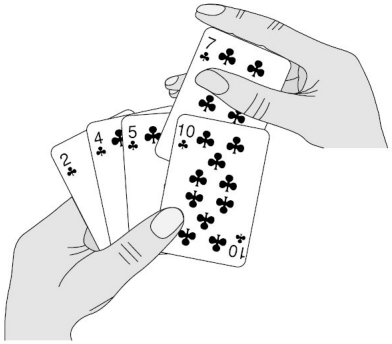

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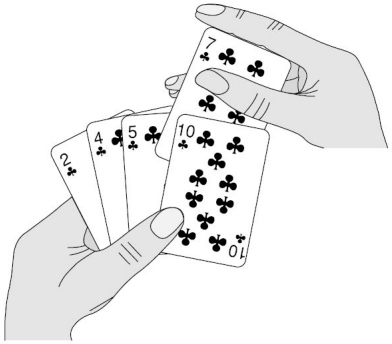

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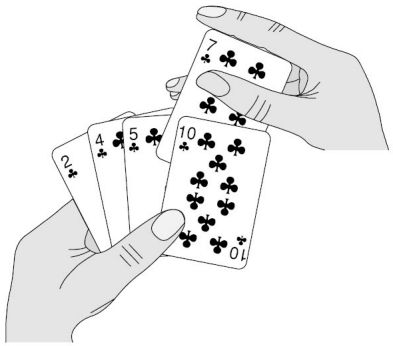

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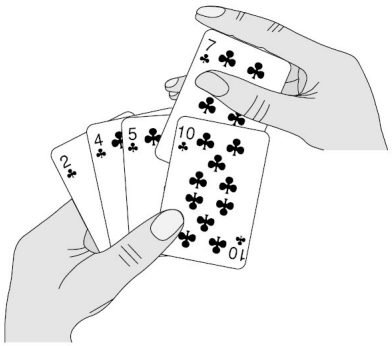

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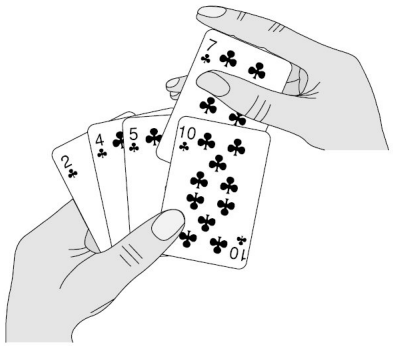

- How do you sort a hand of cards?
- Some common approaches:
  - Take the largest card and move to the end. Repeat with next largest... BubbleSort
  - Start a new list and insert each card into it to keep in order...InsertionSort
  - Divide the cards in half. Sort each half and merge sorted results together....MergeSort
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  ```
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  for j in range(1,len(a)):
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- *Python Tutor demo*
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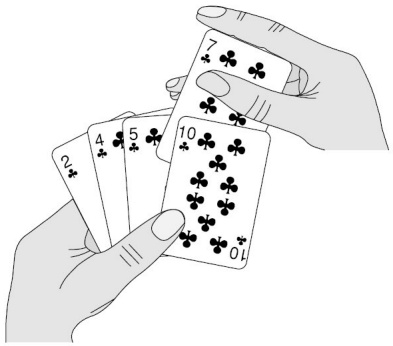
Insertion Sort



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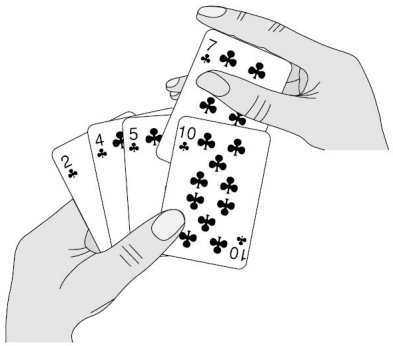

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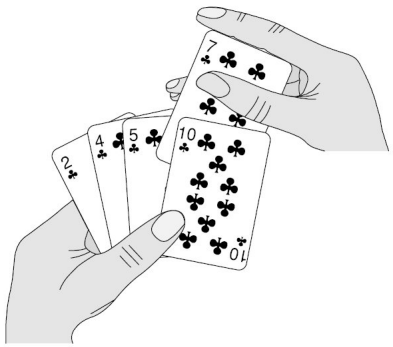

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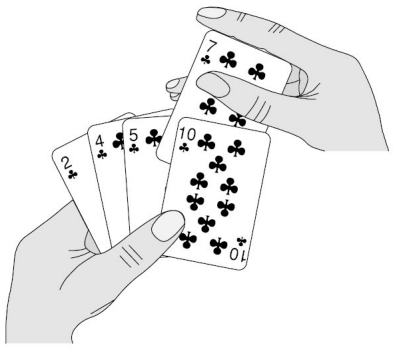

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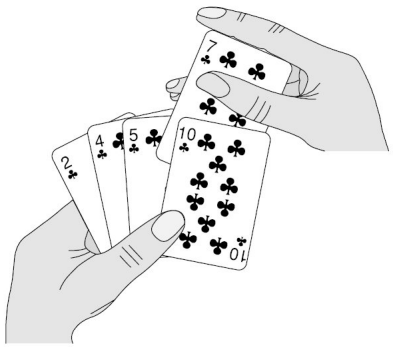

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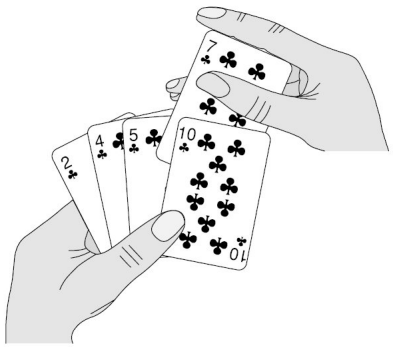

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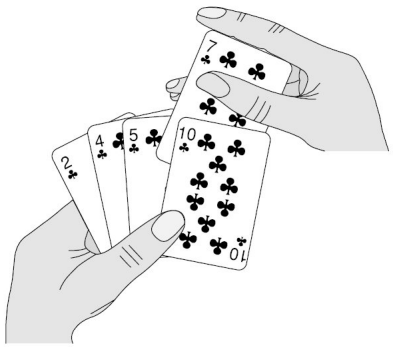

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- *wiki demo*

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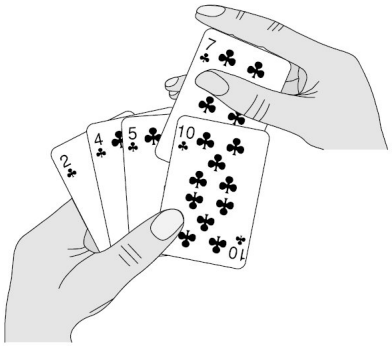

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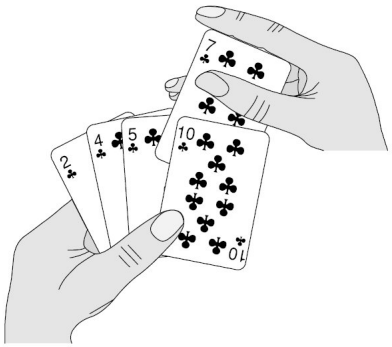

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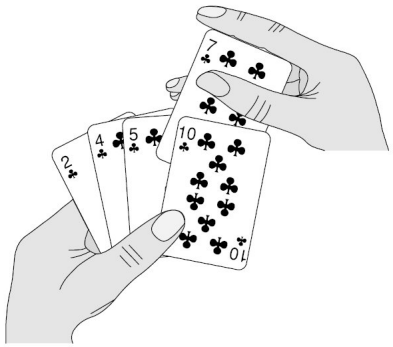

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          j = i
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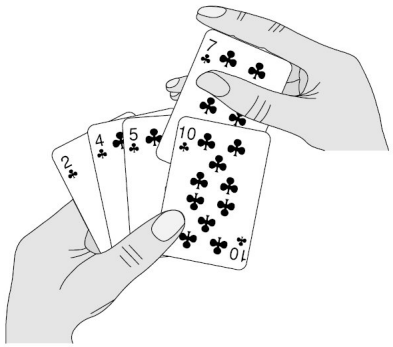

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- *PythonTutor demo*
  ```

In Pairs

In pairs/triples, work through

1. *Problem Solving with Algorithms and Data Structures*, BubbleSort chapter (link on webpage)
2. *Problem Solving with Algorithms and Data Structures*, InsertionSort chapter (link on webpage)
3. If time, *Problem Solving with Algorithms and Data Structures*, MergeSort chapter (link on webpage)

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- We did the "merge" in pairs.
- *ProblemSolving book's demo*

Break

Comparing Algorithms

- Measure the size of the problem, usually called n.

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- Different approaches can take different amounts of time.
- How long does the algorithm take proportional to n?
- *Sorting Algorithms demo*

  Not in demo is the built-in Python sort: timSort (invented by Tim Peters in 2002) that is hybrid of merge sort and insertion sort.

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- *Sorting Demo*

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- The sorting algorithms vary in running time, depending on number of elements and type of data.
- *Sorting Demo*
- Thinking about the worst-case, how many operations are performed in bubbleSort?

Analyzing Sorts by Running Times

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(For big-Oh notation, drop constants and keep only largest terms.)

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- In many programming languages, functions can call themselves.
- This is called recursion (versus iteration).
- Can be a bit confusing to trace through, but Python treats like any other function call.

In Pairs

In pairs/triples, work out (and then try at the shell, `nested.py` on webpage):

1. What does the following do:

```
def nested(t, k):
    for i in range(3):
        t.left(120)
        t.forward(k)
    if k > 10:
        nested(t, k/2)
tess = turtle.Turtle()
tess.shape("turtle")
nested(tess, 320)
```

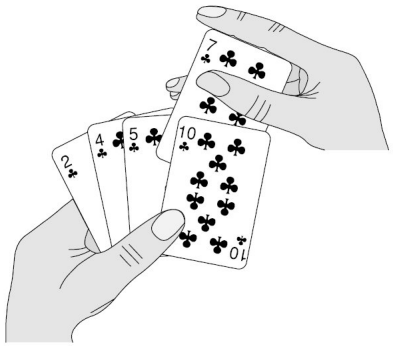
2. Modify the program to draw nested squares.

3. What does the following do:

```
def nested2(t, k):
    for i in range(3):
        t.left(120)
        t.forward(k)
        if k > 10:
            nested2(t, k/2)
tess = turtle.Turtle()
tess.shape("turtle")
nested2(tess, 320)
```

4. Modify the program to draw nested squares.

Recap



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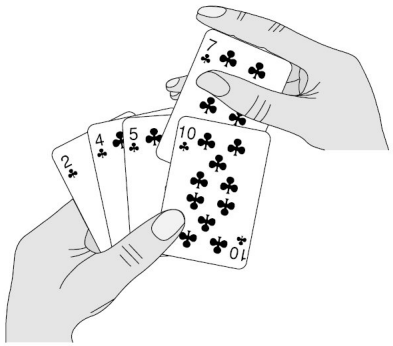

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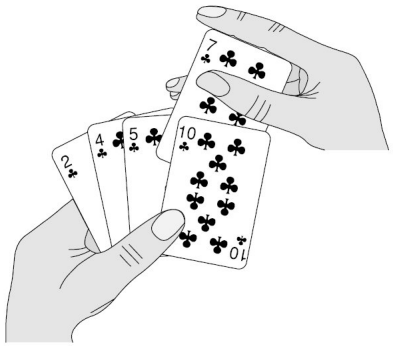

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- Challenges available at `rosalind.info`