Final Examination
Computer Science 761
Lehman College of CUNY
Tuesday, 17 December 2002

NAME (Printed) ______________________________
NAME (Signed) ______________________________

Please show all your work. Your grade will be based on the work shown.

Question 1 (10 points)	
Question 2 (10 points)	
Question 3 (10 points)	
Question 4 (10 points)	
Question 5 (15 points)	
Question 6 (15 points)	
Question 7 (15 points)	
Question 8 (15 points)	
TOTAL	

Useful Formulas

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\qquad$ $\sum_{i=1}^{n} x^i = \frac{x^{n+1}-1}{x-1}$

$\sum_{i=1}^{n} \frac{1}{i} = \ln n + O(1)$ $\qquad$ $\sum_{i=0}^{\infty} ix^i = \frac{x}{(1-x)^2}$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ $\qquad$ $\lim_{n\to\infty}(1+\frac{x}{n})^n = e^x$

$n! = \sqrt{2\pi n}(\frac{n}{e})^n(1+\Theta(\frac{1}{n}))$ $\qquad$ $n! = o(n^n)$

$n! = \omega(2^n)$ $\qquad$ $\lg(n!) = \Theta(n \lg n)$

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$ $\qquad$ $(\frac{n}{k})^k \leq \binom{n}{k} \leq (\frac{en}{k})^k$

$E[X] = \sum_x x Pr[X = x]$ $\qquad$ $Var[X] = E[(X - E[X])^2] = E[X^2] - E^2[X]$

1. (a) ___ For every $a, b > 1$, $n^b = o(a^n)$.
   (b) ___ Counting sort takes time $\Omega(n \lg n)$.
   (c) ___ A heap of n elements has height n.
   (d) ___ Greedy algorithms always produce polynomial time algorithms.
   (e) ___ Huffman codes are an example of dynamic programming.
   (f) ___ Kruskal's and Prim's algoritms both find the minimum flow across a network.
   (g) ___ A directed graph always has more edges than vertices.
   (h) ___ Deciding if a graph contains a minimum spanning tree is NP-complete.
   (i) ___ Given a flow network $G = (V, E)$ with flow f. The flow along a path from $u \in V$ to $v \in V$ is the sum of the flows of the edges in the path.
   (j) ___ The fastest algorithm for multiplying two $n \times n$ matrices is $\Omega(n^3)$.

2. Order the following functions according to their order of growth:

$$2, 2^n, n, \lg n, \lg^{1/2} n, n!, n^2, n^n, 2n, n \lg n$$

   (that is, reorder the list above so that each element is asymptotic bounded above by the next element in the list):

3. Demonstrate the insertion of the keys 5, 28, 19, 15, 20, 33, 12, 17, 10 into a hash table with collisions resolved by chaining. Let the table have 9 slots and let the hash function be $h(k) = k \bmod 9$.

4. Solve the following recurrences:

   (a) $T(n) = 7T(n/2) + \Theta(n^2)$.

   (b) $T(n) = 8T(n/2) + \Theta(n^2)$.

5. (a) Define the term **heap**.

   (b) Build a heap inputting elements in the following order:

$$\{3, 5, 6, 10, 12, 5, 8, 7, 1, 2, 4, 9, 11\}$$

   .

   (c) What is the height of the heap you built for part b)? ______

   (d) Give the **heapsort** algorithm.

6. (a) Suppose that we have numbers between 1 and 1000 in a binary search tree and want to search for the number 301. Which of the following sequences could **not** be the sequence of nodes examined?

   i. 2, 252, 401, 398, 253, 344, 397, 301
   ii. 900, 220, 911, 244, 898, 258, 302, 301
   iii. 925,202, 911, 240, 912, 245, 301
   iv. 2, 399, 387, 219, 266, 382, 381, 278, 301
   v. 935, 278, 347, 621, 299, 392, 358, 301

   (b) Draw binary search tree of height 3 on the set of keys $\{1, 4, 5, 10, 16, 17, 21\}$.

   (c) Write a function that will return the object whose key is minimum in a binary search tree. You may assume that an empty tree has value NIL.

7. (a) What is the smallest number of edges needed to have a cycle in a graph G = (V, E)?

   (b) Draw a graph with 4 vertices that does **not** contain a Hamiltonian cycle:

   (c) Draw a graph with 5 vertices that does contain a Hamiltonian cycle:

8. (a) Write an efficient algorithm to search a graph G = (V,E).

(b) Analyse the running time of your algorithm for both an adjacency list and adjacency matrix representation of the graph G. Do they have the same running time? Why or why not?