Exam 2
Computer Science 761
Lehman College of CUNY
Thursday, 21 November 2002

NAME (Printed) _____
NAME (Signed) _____

Please show all your work. Your grade will be based on the work shown.

Question 1 (5 points)	
Question 2 (15 points)	
Question 3 (10 points)	
Question 4 (10 points)	
Question 5 (10 points)	
Question 6 (10 points)	
Question 7 (20 points)	
Question 8 (20 points)	
TOTAL	

Useful Formulas

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} x^i = \frac{x^{n+1}-1}{x-1}$

$\sum_{i=1}^{n} \frac{1}{i} = \ln n + O(1)$

$\sum_{i=0}^{\infty} ix^i = \frac{x}{(1-x)^2}$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

$\lim_{n\to\infty}(1 + \frac{x}{n})^n = e^x$

$n! = \sqrt{2\pi n}(\frac{n}{e})^n(1 + \Theta(\frac{1}{n}))$

$n! = o(n^n)$

$n! = \omega(2^n)$

$\lg(n!) = \Theta(n \lg n)$

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$(\frac{n}{k})^k \leq \binom{n}{k} \leq (\frac{en}{k})^k$

$E[X] = \sum_x xPr[X = x]$

$Var[X] = E[(X - E[X])^2]$
$= E[X^2] - E^2[X]$

1. (a) ___ In a flow network, the flow on an edge is always the same as the capacity of the edge.

   (b) ___ A directed graph always has more edges than vertices

   (c) ___ Dynamic programming is a form of a greedy algorithms.

   (d) ___ Every undirected graph has at most one minimum spanning tree.

   (e) ___ Binary search trees are directed, acyclic graphs with special properties to make searching efficient.

2. (a) How many different minimal spanning trees are there on a complete graph on 4 vertices with edge weights 1
   (that is, the graph $G = (\{1, 2, 3, 4\}, \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$ and the weight of every edge is 1.)

   _____

   (b) What is the most number of edges possible in a directed graph $G = (V, E)$ where n is the number of vertices?

   _____

   (c) What is the most number of edges possible in a undirected graph $G = (V, E)$ where n is the number of vertices?

   _____

   (d) In a flow network, what is the capacity of a path, p, if the capacity of each edge (u, v) is $c(u, v)$?

   _____

   (e) How many edges are there in a tree on n vertices?

   _____

3. Given the following character set and the frequency each letter occurs, construct a Huffman code for the character set:

	a	b	c	d	e	f
Frequency (in thousands)	5	11	1	4	9	1

4. Topologically sort the following acyclic directed graph. Use the edge relation given by the following table, first draw it, then sort it.

Adjacency Matrix:

	socks	shoes	hat	gloves	coat	watch
socks	0	1	0	0	1	0
shoes	0	0	0	0	1	0
hat	0	0	0	0	0	0
gloves	0	0	0	0	0	0
coat	0	0	1	1	0	0
watch	0	0	0	1	0	0

5. (a) Write Prim's Algorithm.

(b) Write Kruskal's Algorithm

(c) Give a simple example of a graph where the two algorithms will produce different answers.

6. Analyze the following algorithm. You may assume that all flows are integer-valued (that is, $f[u, v]$ is always an integer) and $|f^*|$ is the value of the maximal flow.

```
G is a directed graph with s and t vertices in G.

FORD-FULKERSON(G,s,t)
1  for each edge (u,v) in E[G]
2     do f[u,v] <- 0
3        f[v,u] <- 0
4  while there exists a path p from s to t in the
         residual network G_f
5     do c_f(p) <- min {c_f(u,v) : (u,v) is in p}
6        for each edge (u,v) in p
7           do f[u,v] <- f[u,v] + c_f(p)
8              f[v,u] <- - f[u,v]
```

7. Describe a greedy algorithm to make change consisting of quarters, dimes, nickels, and pennies. Show that your algorithm is optimal.

8. Give an algorithm that determines whether or not a given undirected graph $G = (V, E)$ contains a cycle. Your algorithm should run in $O(V)$ time, independent of the size of E.